Solutions Mechanics & Relativity Test 1

Exam taken on September 15th 2020

True/False Problems

1. You're in a spaceship that takes you to Aldebaran and back to Earth, at constant velocity. At all times during your journey, a reference frame centered on the spaceship is inertial.
   Correct answer: False
   Explanation: The spaceship can be considered to be an inertial reference frame on its journey from the Earth to Aldebaran, and then again on its journey from Aldebaran and the Earth. However, at the moment it turns around to head back, its direction changes and thus velocity changes. This means it's velocity cannot be considered to be constant throughout its journey and it isn't an inertial reference frame.

   Practically speaking, it's speed itself would also change as it turns around, as it must decelerate, stop for a moment and then accelerate again to reach the constant velocity it was travelling at during its journey.

2. Suppose the Galilean transformation equations are true and you are in a spaceship traveling through the stars, at three times the speed of light relative to the ether. You point a laser beam backwards (wrt your direction of motion) and slightly to your left. The laser beam appears to be pointing forward left when you look at it.
   Correct answer: False
   Explanation: If the laser is pointed towards the back left direction, as your spaceship moves forward (in the +x direction) at speed 3c, it's light would appear bent even further back - further towards the -x direction. In other words, the laser beam would appear to be angled closer to the trajectory of the spaceship. This is a consequence of the spaceship moving 3 times as fast as the photons emitted from the laser.

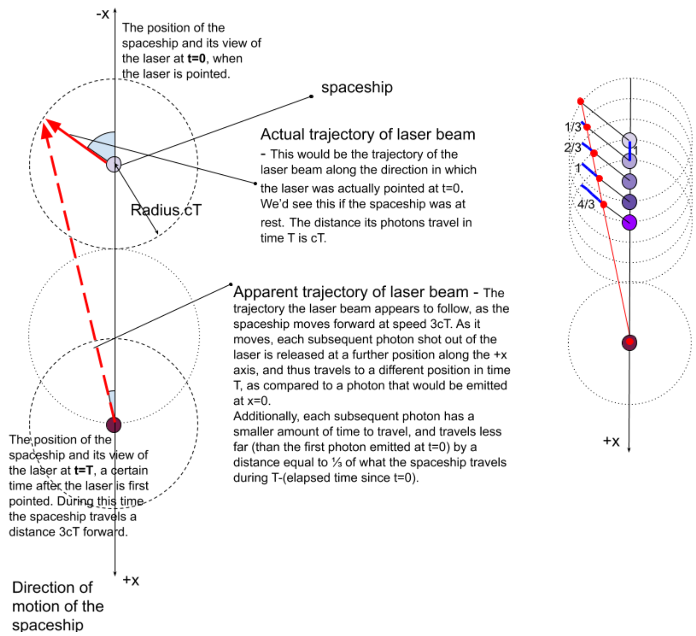


Figure 1 - Image for Question 2

3. Suppose the Galilean transformation equations are true and you are in a spaceship traveling through the stars, at three times the speed of light relative to the ether. Your only field of view would be in front of you, inside a cone angled at roughly 19.5 degrees to your direction of motion.

Correct answer: True

Explanation: As your spaceship travels through the ether, at any moment in time T, all the light inside a sphere of radius cT will be visible to you. As you move forward your sphere moves with you, with the new sphere in each moment in time, overlapping partially with the sphere at the previous moment.

The cone that makes up your field of view is angled such that the furthest extent of your field of view, will lie on the intersection between the cone's circular surface and the surface of your sphere at time T.

$$sin\,\theta = \frac{cT}{3cT} = 1/3 \rightarrow \theta = sin^{-1}(1/3) = 19.47 \approx 19.5\ degrees$$

Ɣs travelling towards the spaceship, emitted from objects located behind the spaceship (at a moment in time), also appear in this conic field of view. These light beams appear bent in the reverse direction to where they were pointed (*i.e.* mirrored in the line perpendicular to the trajectory of the spaceship).

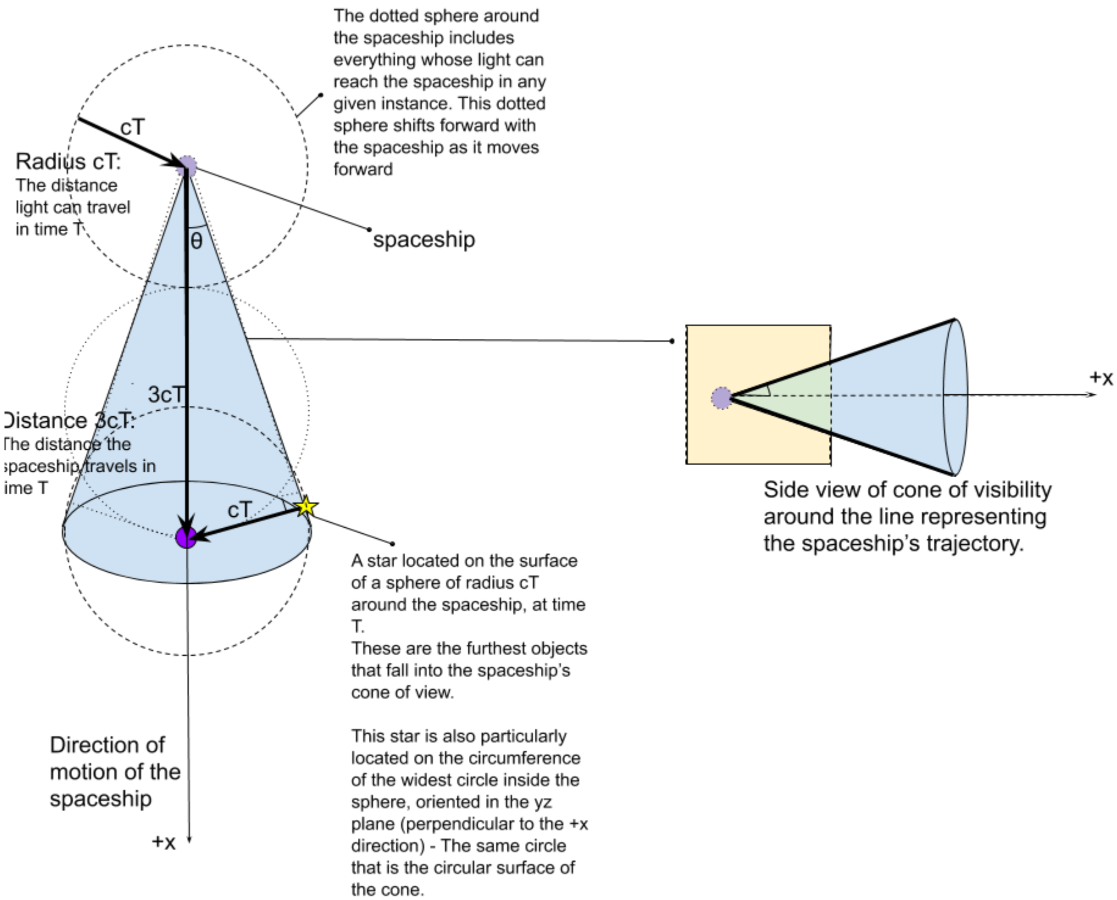


Figure 2 - Image for Question 3

4. Two siblings are at a go-kart park, out to get each other. Sibling A is driving at 60 km/hr at an angle of 30 degrees south of east and Sibling B is driving at 60 km/hr at an angle of 45 degrees west of north towards A. We choose the race track as the Home Frame and kart A as the Other Frame. The relative velocity of kart B from the perspective of sibling A is $[v'_{x,B}, v'_{y,B}] = [-30\sqrt{3} + 30\sqrt{2}, -30 - 30\sqrt{2}]$ km/hr

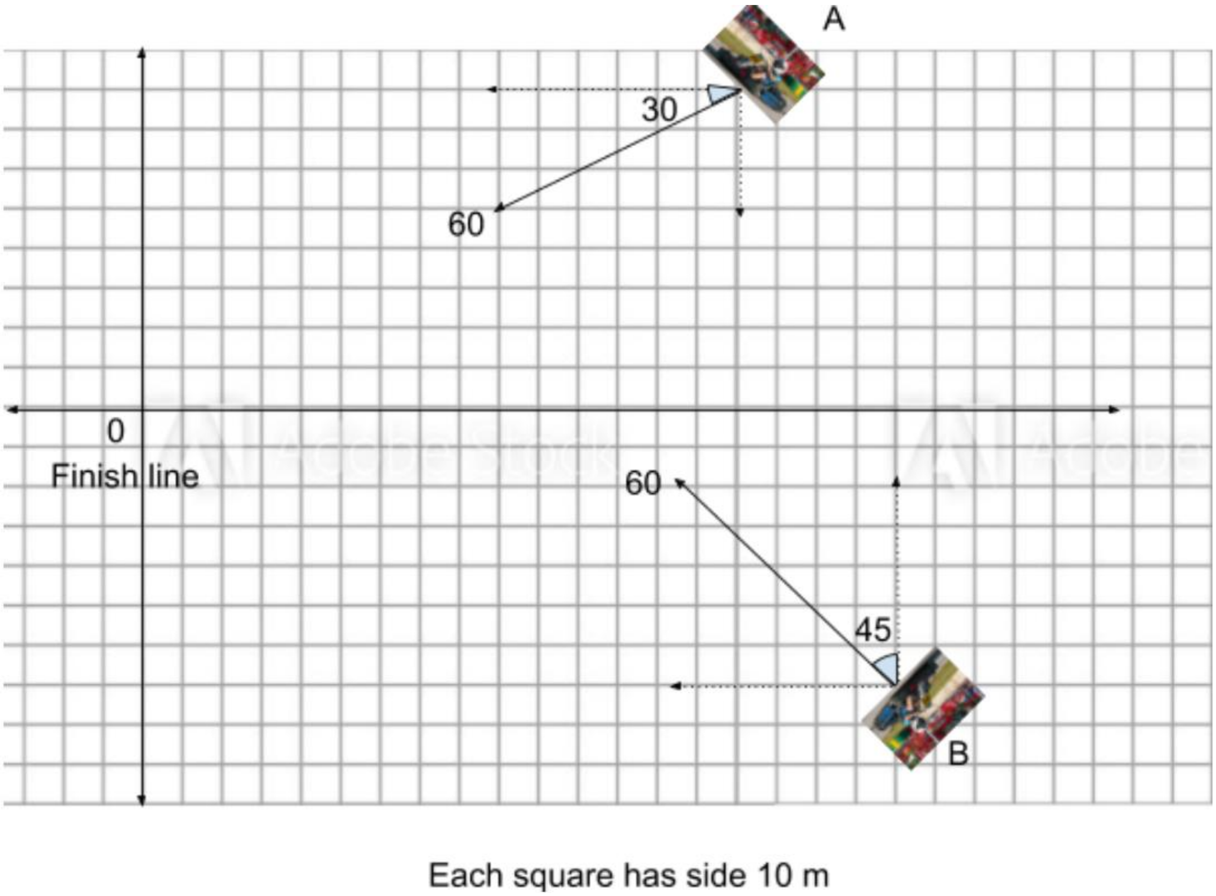


Figure 3 - Image for Question 4

Correct answer: False

Explanation: Home Frame: Race track

Other Frame: kart A

β, the relative velocity of kart A (Other Frame) w.r.t. the race track (Home Frame) is

$\mathbf{v}_A = [v_{x,A}, v_{y,A}] = [-60\cos(30), -60\sin(30)] = [-30\sqrt{3}, -30]$ km/hr

We would like to know the velocity of kart B in the Other Frame $\mathbf{v'}_B$ (*i.e.* from the perspective kart A). We already know the velocity of kart B in the Home Frame $\mathbf{v}_B = [v_{x,B}, v_{y,B}] = [-60\sin(45), 60\cos(45)] = [-30\sqrt{2}, 30\sqrt{2}]$ km/hr

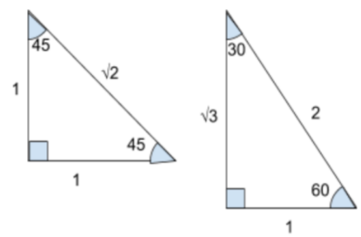
Finally, using the Galilean velocity transformation equation, we get:

$\mathbf{v'}_B = \mathbf{v}_B - \boldsymbol{\beta} = [-30\sqrt{2}, 30\sqrt{2}] - [-30\sqrt{3}, -30] = [-30\sqrt{2} + 30\sqrt{3}, 30\sqrt{2} + 30] = [v'_{x,B}, v'_{y,B}]$

The answer given in the question is the velocity of kart A in the frame of reference of kart B.

Note: You can find exact values of sines and cosines of common angles using right angled triangles with sides of the appropriate ratios.

You should know the values of the sines, cosines and tangents of the below two triangles.



e.g. If one of the remaining angles of a right angled triangle is 45°, so is the third angle. We set an adjacent side to have length 1. Using the law of the sines sinA/a=sinB/b=sinC/c. we can deduce that the opposite side must also have length 1. Finally, the pythagoras theorem gives us that the hypotenuse equals √2. It is then evident that sin(45) = cos(45) =12

5. The choice of a coordinate time does not imply the choice of an entire frame of reference.
Correct answer: False
Explanation: Coordinate time is frame dependent.

6. The coordinate time is always with respect to some observer.
Correct answer: True
Explanation: The interpretation of the coordinate time is dependent on the perspective of the observer.

7. Two events are considered to be simultaneous in a reference frame if the coordinate time is the same for both events.
Correct answer: True
Explanation: An event in one frame will occur simultaneously with an event in another frame if their coordinate times are the same because coordinate time is frame dependent.

8. If the coordinate time is the same for two events, it is likely the two events will not be simultaneous from the viewpoint of another reference frame.
Correct answer: True
Explanation: If different reference frames are used, two events will likely not be simultaneous even though the coordinate time may be the same for both events.

9. 'kg' is an SR unit.
Correct answer: True
Explanation: SR units are almost the same as SI units except for the fact that they change the SI unit of distance to a unit of time. 'kg' therefore remains the same and is an SR unit as well as an SI unit.

10. The SR unit 's' can belong to more than one physical quantity.
Correct answer: True
Explanation: SR units are almost the same as SI units except for the fact that they change the SI unit of distance to a unit of time. Distance and time therefore have the same unit, which can be 's'. 's' can therefore belong to more than one physical quantity.

11. A frame moving with an acceleration of 3 m/s^2 is an inertial frame.
Correct answer: False
Explanation: Inertial frames move with a constant velocity, so no acceleration. Since the frame in the question has a nonzero acceleration, it is not an inertial frame.

12. A spaceship is moving with a constant velocity away from an observer on Earth. Their clocks are synchronized and when the spaceship reaches its destination, it signals with a laser to the observer on Earth. The observer on Earth will see the same time on the clock when the signal was received as the crew of the spaceship saw when it was sent.
Correct answer: False
Explanation: Since light takes some time to travel, the time at which the signal was sent is not the same as the time at which it was received.

13. Consider a collision between two particles. Bob and Alice are in two different inertial frames moving relative to each other. Both measure the total momentum before the collision. Since the laws of physics are the same in all inertial frames, Bob and Alice should find the exact same result for the total momentum before the collision.
Correct answer: False
Explanation: There is no physical law that conserves 'pre-collision momentum' – this is only a part of another physical law. There is only a law that conserves the total momentum over a collision, and this will also be the case in this collision.

14. Coulomb's formula for the net force between two point charges is a law of physics. Therefore, it holds in all inertial reference frames.
Correct answer: True
Explanation: The laws of physics are the same in every reference frame. The force exerted on the two particles will be the same in the two reference frames.

15. In developing the theory of special relativity, Einstein assumed Maxwell's equations and the principle of relativity were true. These assumptions inevitably led to the conclusion that the speed of light must be the same in all inertial reference frames.
Correct answer: True
Explanation: There would be paradoxes in the laws of physics if one could catch up with a light beam, for example in the scheme of causality.

16. The principle of relativity is a postulate (pre-supposed rule) of special relativity. Therefore, it cannot be proven by experiments, only disproven.
Correct answer: True
Explanation: If you build a theory around some axioms, you can never disprove the fundaments of your theory. You start off with 'given that this is true, what will be the results of experiment X?' Then your hypotheses will never contradict the axioms.

17. Two observers have carefully synchronized their clocks. They will only agree that the others' clocks are properly synchronized if they're not moving relative to each other.
Correct answer: True
Explanation: Two clocks are synchronized if they measure the speed of a light signal traveling between them to be c. If the observers are moving relative to each other, they would see each other's clocks move and conclude that they're not synchronized.

18. Consider a conventional spacetime diagram and an object A that moves in the +x direction. As object A is moving, its worldline in the diagram curves upwards. This means that object A has a positive acceleration.
Correct answer: False
Explanation: The speed of an object is the inverse of the slope of its worldline. Since the worldline of object A curves upwards, the slope is increasing. Therefore the speed is decreasing. I.e. the acceleration is negative.

19. Alice measures the coordinates of two events A and B. She finds that they occurred simultaneously and that $x_A = -x_B = 2s$. Bob moves in the -x direction relative to Alice. This means that Bob will measure event B to occur after event A.
Correct answer: False
Explanation: Bob will see Alice to be moving in the +x direction. This means that in Bob's frame, a radar signal would take longer to reach event A than event B. See figure R2.8 of the book (but swap events A and B).

20. A student determines the kinetic energy of a train to be 3 kg in SR units. To find the kinetic energy in Joules, they must multiply this number by c.
Correct answer: False
Explanation: Joules contains m^2. This means that they must multiply by c^2.

21. A frame moving along the trajectory of a proton in a linear collider can be an inertial frame.
Correct answer: False
Explanation: The particle is still accelerating, hence it can not be an inertial frame according to SR.

22. Special Relativity relies on the presence of an ether.
Correct answer: False
Explanation: The idea preceded the principle of special relativity and lead Lorentz for formulate laws of relativity which Einstein used to build his theory of relativity without an ether.

23. A worldline connects two events in a spacetime diagram.
Correct answer: True
Explanation: A worldline always connects two events. Not all events are connected by wordlines.

24. Light always moves along 45 degree angles in a space-time diagram.
Correct answer: True
Explanation: The speed of light is constant and frame invariant and moves with 45 degrees in a spacetime diagram

25. A Galilean transformation does not affect the time-coordinate.
Correct answer: True
Explanation: Galilean transformations only affect spatial coordinates along the line of propagation.

26. The red car and the blue car are having a race. Their worldlines are shown on the diagram above. At time t1, the blue car is going faster than the red car.
Correct answer: False
Explanation: We can see on the diagram that the blue worldline has a steeper slope than the red worldline at the time t1. A steeper slope means that the object is moving more slowly.

27. The average height of a Dutch man is approximately 6 ns in SR units.
Correct answer: True
Explanation: 1 ns is approximately 1 foot, so 6 ns is approximately 6 feet, which is 183 cm. This is the average height of a Dutch man. If you don't want to deal with heathen units, 1 ns is about 30 cm. This makes 6 ns about 180 cm, which is very close to the average height of a Dutch man, and certainly closer than 5 ns or 7 ns would be.

28. The SR unit for force is s^{-1}
Correct answer: False
Explanation: Force = mass × acceleration. This means that when converting to SR units you have to have kg in your units. The actual SR unit for force is kg s^{-1}.

29. To calculate the velocity of an object from a spacetime diagram, you simply take the slope of its worldline.
Correct answer: False
Explanation: Because the slope of a line is Δy/Δx, and on a spacetime diagram time is on the y-axis, the slope of an object's worldline would be Δt/Δx. This actually gives you the inverse of the object's velocity, so to find the velocity of an object you take the inverse of the slope of the worldline (1/slope)

30. Assume we have a particle at rest in a lab. At t=0 the particle decays into particle C and particle D, which shoot away in opposing directions. If we take the Lab as our Home Frame and particle D as our Other Frame we will be dealing with solely positive velocities for beta and v'C.
Correct answer: False

Explanation: Since we take the lab frame as home frame, one particle will shoot away in the positive x-direction, while the other soots away in the negative x-direction, hence the velocity of one of the particles will be negative.

31. If the Other Frame is moving in the -x-direction as seen from the Home Frame, the location of an object in the home frame is described by: $x = x' + \beta t$
Correct answer: True
Explanation: The value of beta will be negative, but since there are no absolute value bars around the beta, the standard equation applies.

32. Suppose that the Galilean concept of relativity were valid. Two observers, each in a different inertial frame, synchronize their clocks. They observe a firecracker going off. One of the observers is at rest from the firecracker's frame of reference, while the other is not. Both observers observe the firecracker going off at the same time.
Correct answer: True
Explanation: since we are treating a Galilean situation, the relativistic effects don't play a role, hence we observe the firecrackers to go off simultaneously

33. Suppose that the Galilean concept of relativity were valid. Two observers, both in a different inertial frame, synchronize their clocks. They see a firecracker going off. One of the observers is at rest from the firecracker's frame of reference, while the other is not. Both observers see the firecracker going off at the same time.
Correct answer: False
Explanation: since we are specifying seeing as the light from the event reaching our eye, the light will have to travel a different distance for the observer that is moving compared to the observer at rest. Hence the light will reach the observers at different times.

34. 1 kg of energy is approximately 10^18 J
Correct answer: False
Explanation: The answer is more than a factor of 10 different, hence it is not approximately the same.

35. This diagram is according to the conventions from the "Six Ideas That Shaped Physics" book.

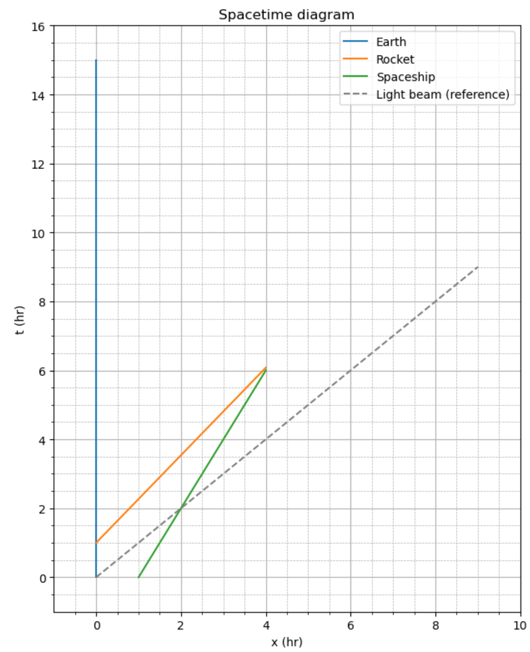


Figure 4 - Image for Question 35

Correct answer: True

Explanation: The axis are labelled and have units, the home frame is picked such that the other frame is moving in the positive x direction and beta is defined to be positive. The diagram is drawn such that no objects have a slope smaller than a photon, which means that nothing travels faster than the speed of light.

36. The spaceship in this diagram travels faster than light.

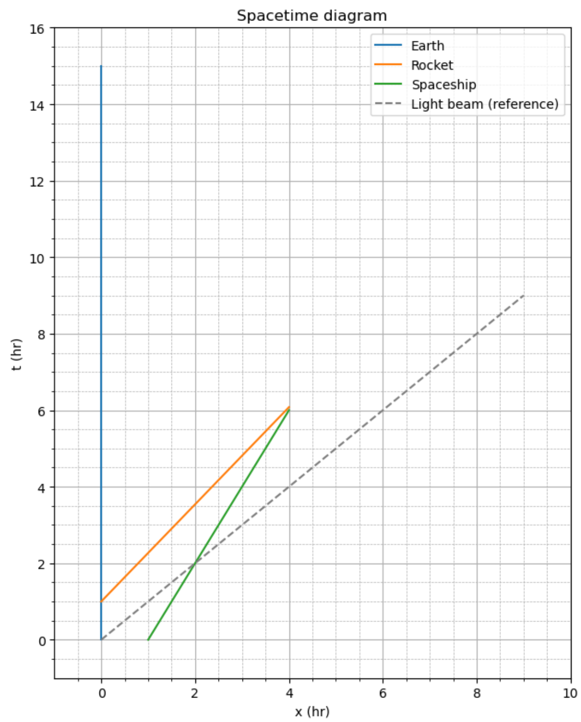


Figure 5 - Image for Question 36

Correct answer: False.

Explanation: The slope of the light beam is 1, which is the speed of light. The rocket, spaceship, and earth all have a larger slope, which means that their velocity is smaller (remember that the inverse of the slope is proportional to the velocity, which is different from when the t-axis is the horizontal one and x is the vertical one, like you often had in high school.)

37. Take the earth to be the Home Frame and the spaceship in this diagram to be the Other Frame.

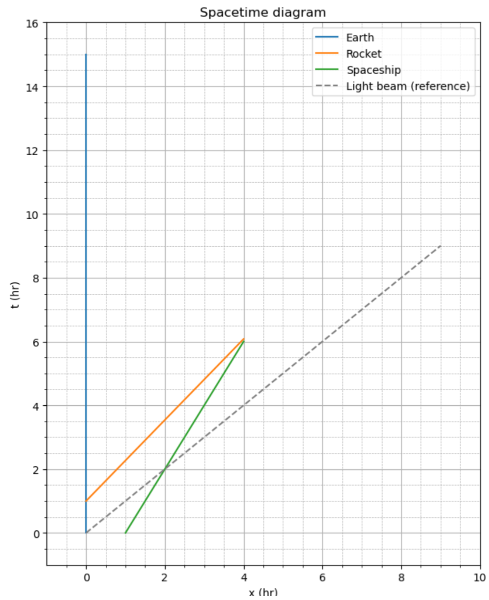


Figure 6 - Image for Question 37

Correct answer: True

Explanation: Because the other frame is moving in the positive x-direction.

38. Take the rocket to be the Home Frame and the spaceship in this diagram to be the Other Frame.

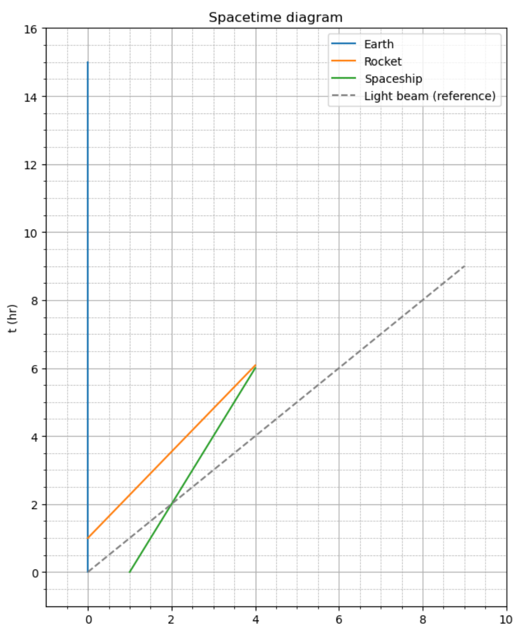


Figure 7 - Image for Question 38

Correct answer: False

Explanation: In the rocket's frame, the spaceship is getting closer while it was some distance away on the right. If we would continue drawing the lines of the objects in the diagram, the spaceship would even show up on the left of the rocket; the sign of beta is hence negative, as if beta were in the positive x-direction it would be positive.

39. Assume we have a particle at rest in a lab. At t=0 the particle decays into particle C and particle D, which shoot away in opposing directions. If we take the Lab as our Home Frame and particle D as our Other Frame we will be dealing with solely positive velocities for beta and v'C.
Correct answer: False
Explanation: since we take the lab frame as home frame, one particle will shoot away in the positive x-direction, while the other soots away in the negative x-direction, hence the velocity of one of the particles will be negative.

40. If the Other Frame is moving in the -x-direction as seen from the Home Frame, the location of an object in the home frame is described by: $x = x' + \beta t$
Correct answer: True
Explanation: The value of beta will be negative, but since there are no absolute value bars around the beta, the standard equation applies.

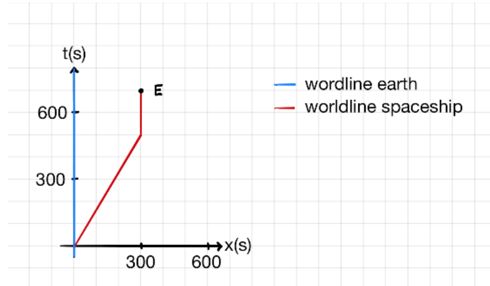
41. Suppose that the Galilean concept of relativity were valid. Two observers, each in a different inertial frame, synchronize their clocks. They observe a firecracker going off. One of the observers is at rest from the firecracker's frame of reference, while the other is not. Both observers observe the firecracker going off at the same time.
Correct answer: True
Explanation: Since we are treating a Galilean situation, the relativistic effects don't play a role, hence we observe the firecrackers to go off simultaneously.

42. Suppose that the Galilean concept of relativity were valid. Two observers, both in a different inertial frame, synchronize their clocks. They see a firecracker going off. One of the observers is at rest from the firecracker's frame of reference, while the other is not. Both observers see the firecracker going off at the same time.
Correct answer: False
Explanation: Since we are specifying seeing as the light from the event reaching our eye, the light will have to travel a different distance for the observer that is moving compared to the observer at rest. Hence the light will reach the observers at different times.

43. 1 kg of energy is approximately 10^18 J
Correct answer: False
Explanation: the answer is more than a factor of 10 different, hence it is not approximately the same.

Multiple choice problems

1. The spacetime diagram below shows the worldline of a spaceship that leaves earth, travels for a certain amount of time, then comes to rest. Eventually the rear end of the spaceship explodes (event E). Hint: draw the spacetime diagram for yourself.



Correct answer: b.

Explanation: The speed of the spaceship is given by the inverse of the slope of the worldline. The slope of the worldline is 5/3, so the speed is 3/5.

2. At some point, the spaceship comes to rest. What are the coordinates (t, x) of this event?

Correct answer: d

Explanation: As we see from the diagram, the worldline becomes vertical starting at (t, x) = (500 s, 300 s), indicating the spaceship is at rest from that point onward. These are then also the coordinates at which the spaceship comes to rest.

3. As the rear end of the spaceship explodes, the cockpit module (with astronauts in it) decouples from the rest of the spaceship (forming the spaceship version of a lifeboat). It travels back to earth with a constant speed of ⅔ relative to Earth. At what time is the cockpit module expected to arrive on Earth?

Correct answer: c

Explanation: The cockpit module goes back to earth at speed ⅔, so that the worldline has a slope 3/2 and is directed from event E back to earth (the blue worldline). Drawing the worldline in the diagram (the green worldline), we have:

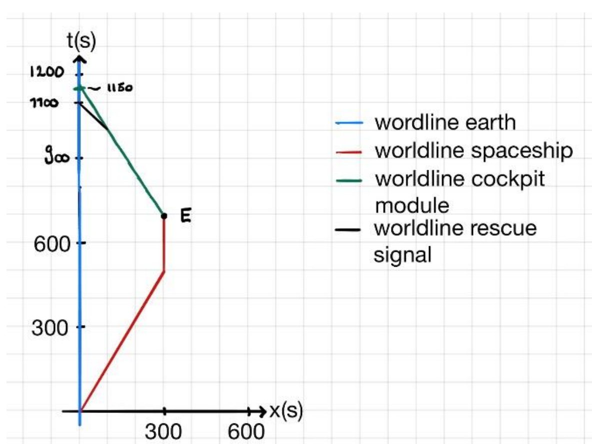


Figure 8 - Image for Question 3

The intersection of the green worldline with the earth-worldline (blue) is at t = 1150 s. (One could also solve this using algebra.)

4. As the cockpit module is a distance of 100 s from earth, its engine suddenly stops working and the module comes to rest. Instantly, a rescue signal is sent to Earth. What is the earliest possible moment in time this signal reaches Earth?
Correct answer: a
Explanation: See black worldline in diagram of 3.) The fastest possible rescue signal is a signal moving at the speed of light, so its corresponding worldline would have a slope of one. As this signal is emitted when ⅔ of journey of the cockpit module to earth is completed, it is emitted by the cockpit module at x = 100 s. Drawing the quickest worldline of the rescue signal (see black line in diagram of 3.) and examining its intersection with the earth-worldline, we find that the rescue signal reaches the earth at t = 1100 s at the earliest.

Introduction for problem 5-7: he five situations below represent the movement of passengers (circles) on trains (rectangles), with the direction of their speeds indicated by the arrows. The magnitudes are written next to the arrows.

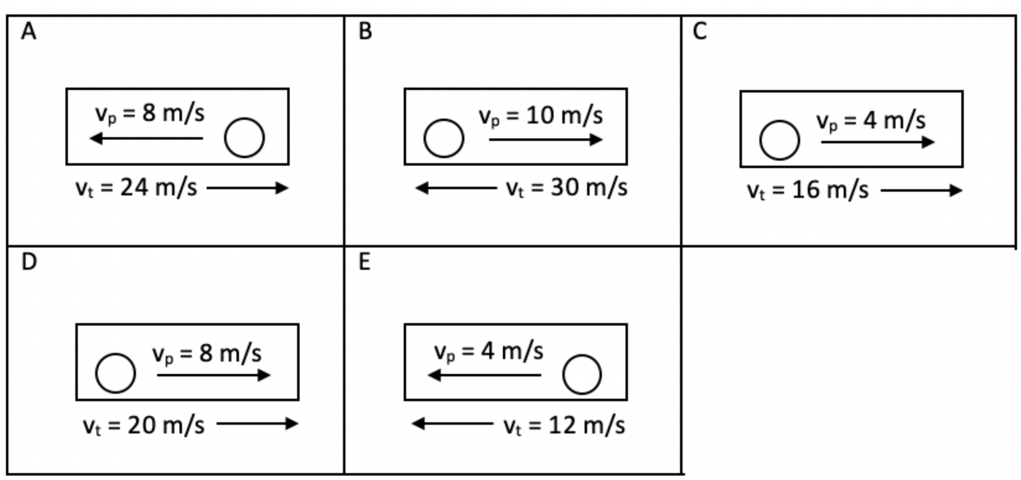


Figure 9 - Image for Question 5-7

An observer is standing beside the train track watching the train go by.

Notes: I define left as negative and right as positive. The observer (HOME FRAME) sees the following net speeds for each situation:

A → | - 8 m/s + 24 m/s | = 16 m/s
B → | 10 m/s - 30 m/s | = 20 m/s
C → | 4 m/s + 16 m/s | = 20 m/s
D → | 8 m/s + 20 m/s | = 28 m/s
E → | -4 m/s - 12 m/s | = 16 m/s

5. In which situation would the observer see the passenger moving the fastest?
Correct answer: D.
Explanation: | 8 m/s + 20 m/s | = 28 m/s

6. In which situation would the observer see the passenger moving the slowest?
Correct answer: A and E.
Explanation: A → | - 8 m/s + 24 m/s | = 16 m/s. E → | -4 m/s - 12 m/s | = 16 m/s

7. in which situations would the observer see the passengers moving at the same speed?
Correct answer: B and C.
Explanation: B → | 10 m/s - 30 m/s | = 20 m/s. C → | 4 m/s + 16 m/s | = 20 m/s

8. Brian is standing in a train moving at a velocity v from left to right relative to Kraig, who is standing on the track outside the train. As Brian passes Kraig, Brian drops a bowling ball out of the train's window:

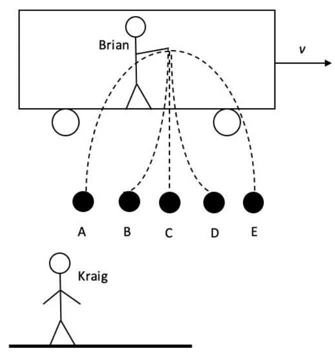


Figure 10 - Image for Question 8

Ignoring air resistance, what path of the bowling ball would Kraig observe?
a. A
b. B
c. C
d. D
e. E

Correct answer: E

Explanation: When Brian released the bowling ball, Kraig would observe it moving at a constant velocity to the right (OTHER FRAME) *and* increasing velocity downwards (due to gravity).

9. Brian is standing in a train moving at a velocity v from left to right relative to Kraig, who is standing on the track outside the train. As Brian passes Kraig, Brian drops a bowling ball out of the train's window:

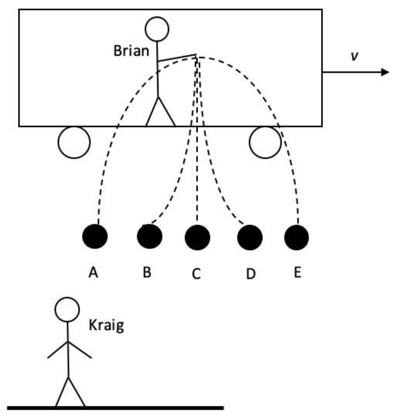


Figure 11 - Image for Question 9

Ignoring air resistance, what path of the bowling ball would Brian observe?
a. A
b. B
c. C
d. D
e. E

Correct answer: C
Explanation: When Brian released the bowling ball, he would observe it moving straight downward with an increasing velocity (due to gravity).

10. Consider two billiard balls A and B that only move along the x-axis. In a certain inertial reference frame (the home frame), ball A has a mass of 3 kg and a speed of -3 m/s and ball B has a mass of 5 kg and a speed of 5 m/s. The so-called center-of-mass frame has the property that the total momentum of the balls, as measured in this frame, is 0. If we consider this to be the other frame, what is β?
a. -7 m/s
b. 0 m/s
c. 2 m/s
d. 7 m/s
e. -2 m/s

Correct answer: C

Explanation: According to Galilean relativity v' = v - β. The total momentum in the other frame is given by p' = m1v1' + m2v2' = m1(v1 - β) + m2(v2 - β) = 0. Solving for β gives β = (m1v1 + m2v2)/(m1 + m2) = 2 m/s.

11. A passenger inside a moving train throws a ball towards the front of the train. A person on the ground looks at the train and measures the ball to have landed after two seconds and traveled a distance of 100 m. The passenger, however, claims the ball only traveled a distance of 10 m before landing. How fast is the train moving relative to the ground?
a. 70 m/s
b. 45 m/s
c. 60 m/s
d. 15 m/s
e. 40 m/s

Correct answer: B
Explanation: Let the ground be the home frame and the train the other frame. And let the direction in which the train is moving define the +x direction. According to Galilean relativity the distances that the observers measure are related by $\Delta x' = \Delta x - \beta \Delta t$. This gives $\beta = \frac{\Delta x - \Delta x'}{\Delta t} = \frac{100-10}{2} = 45\ m/s$.

12. At time t = 0, a radar installation on Earth sees an unknown spaceship flying by that moves at β = 0.5. When the spaceship is at x = 2 s, the radar installation emits a radio signal towards the spaceship. When the signal reaches the spaceship, they send another signal back. When does the radar installation receive this signal? Choose a range. (Hint: You might find it helpful to draw a spacetime diagram.):
a. 2s – 4s
b. 5s – 7s
c. 8s – 10s
d. 11s – 13s
e. 14s – 16s

Correct answer: D
Explanation: The spaceship reached x = 2s at t = x/β = 4s. Light travels at a speed of 1, so the relative speed (as measured by the radar installation) between the spaceship and the signal is u = 1 - β = 0.5. This means that the time it took the signal to reach the spaceship is x/u = 4s. Again, light travels at a speed of 1, so this also happened at x = 4s. Then sending the signal back takes yet another 4s. Adding everything up gives a total time of 4 * 3 = 12s.

13. Consider a train that moves away from an observer on the ground at a speed β = 0.25. A passenger in the train sends two light flashes, one after the other, towards the observer. The observer *measures* 8 seconds between the two flashes. According to the observer, how much time passes between *seeing* the two flashes. Choose a range. (Hint: You might find it helpful to draw a spacetime diagram.):

a. 15s – 17s
b. 12s – 14s
c. 9s – 11s
d. 6s – 8s
e. 3s – 5s

Correct answer: C
Explanation: In between sending the two light flashes, the train travels a distance of 0.25 * 8 = 2s. This means that the second light flash needs to travel a distance of 2s more to reach the observer than the first. Therefore the second light flash reaches the observer 2 + 8 = 10s later than the first.

Introduction for question 14-17: In the space-time diagram below, each tick represents 1 light year (ly) and the origin is where the x and t axes cross. The starship 'Enterprise (E) moves with a velocity v = -0.5. On planet Zoros (A), the inhabitants send out a distress signal in all directions.

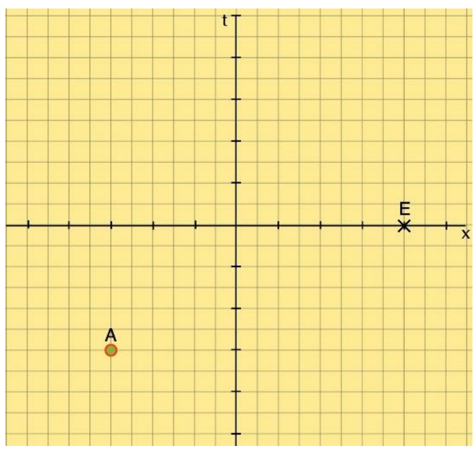


Figure 12 - Image for Question 14

14. How many years (in coordinate time) does the signal sent out from Zoros take before it reaches the starship 'Enterprise'? (Hint: you may find it useful to draw a spacetime diagram.)
a. 5-6 ly
b. 3-4 ly
c. 6-7 ly
d. never
e. 2-3 ly

Answer: a
Explanation: There are several ways to solve this problem. The easiest way is to draw a spacetime diagram. First, you put the proper labels on the axis. See figure below. Draw the signal, which propagates at the speed of light (slope of 1). The spaceship is moving left in the diagram (with speed 0.5, or slope of 1/0.5 = 2). The two lines intersect between 2-3 ly

on the coordinate axis. The total time passed since the emission of the signal is then 3 + (2-3) = 5-6 ly.

To get the exact answer, use linear algebra. The signal moves as $t = x$. The spaceship moves as $t = -2x+a$. We can determine a from when t = 0 ly, x = 4 ly (in the diagram). Hence the ship is described as $t = -2x+8$. To find the spacetime coordinates just equate these:
$t_{signal} = t_{ship}$ or $x = -2x+8$ or $3x = 8$ ly. Solving for x, you find x = 8/3 ly = 2 ⅔ ly. Adding that to the modulus distance of the planet to the origin ($|x_{planet}|=3$ ly) we get the exact distance between the planet and the ship as they intercept: 5 ⅔ ly. Since t = x for the signal, the time this happens should also be 5 ⅔ ly .

15. How far away from planet Zoros is the starship 'Enterprise' when it intercepts the distress call? (Hint: you may find it useful to draw a spacetime diagram.)

a. 2-3 ly
b. 5-6 ly
c. 6-7 ly
d. > 7 ly
e. < 2 ly

Correct answer: b
Explanation: See above.

16. Cpt Picard of starship 'Enterprise' immediately realizes the seriousness of the matter. While the hyperdrive is on repair, which prevents the ship from going into warp speed, he is able to use his torpedo thrusters to boost the ship's velocity to 0.8 in the direction of the planet. How long after the inhabitants of planet Zoros send out the distress call does the Enterprise reach the planet? (Hint: you may find it useful to draw a spacetime diagram.)
a. 13-14 ly
b. 12-13 ly
c. 11-12 ly
d. 10-11 ly
e. 9-10 ly

Correct answer: b
Explanation: We can again draw the lines in the diagram and read it off. Draw a line from the point of interception to the left with a slope of 1/beta = 5/4 until the line intersects with the worldline of the planet (vertical line starting at t = -3 ly and x = -3 ly). Reading of the point of intersection we find for the coordinate time 9-10 ly since t = 0 ly. Adding 3 ly from the planet crossing the line t = 0, we find 12-13 ly.

The exact answer can be found again using linear algebra. The spaceship starts moving as $t = -5/4\ x + a$. We can obtain a by realizing that at $x = 2\ ⅔$ ly, $t = 2\ ⅔$ ly . Inserting this we find $2\ ⅔ = -5/4\ (2\ ⅔) + a$. Or $a = 2\ ⅔ \ast (1+5/4) = 6$ ly. So the spaceships worldline can be described as $t = -5/4\ x+6$. Again the planet spatial location is $x_{planet} = -3$ ly, and we have to add this distance modulus. Inserting this into this equation gives $t = 9\ ¾$ ly. This is the time since $t = 0$ ly. We should take the time since $t = -3$ ly, and thus our final answer is that they reach the planet after 12 ¾ ly.

17. Suppose the inhabitants of planet Zoros decide to send a small spaceship with a speed of 0.8 towards the Enterprise after 5 years and 8 months. How many years after sending out the distress call will the ship reach the Enterprise? (Hint: you may find it useful to draw a spacetime diagram.)
a. 10-11 ly
b. 8-9 ly
c. 12-13 ly
d. 9-10 ly
e. 11-12 ly

Correct answer: d
Explanation: You can draw the diagram. 5 years and 8 months is identical to 5 ⅔ ly. Now draw a line with a slope 1/beta = 5/4 to the right. We read that the crossing happens at t = 6-7 ly. This is from the origin. The time past since first emitting the signal gives 9-10 ly.

The exact answer can be obtained by realizing that both ships leave at the same coordinate time $t = 2\ ⅔$. The distance between the planet and the ship is 5 ⅔ ly (question 2). Because they travel at the same speed, they will both cover half of that distance before they meet, so 2 ⅚ ly. How long does it take to travel that distance? Just divide this by their speed, which is ⅘ the speed of light, $\Delta T = 2\ ⅚ / ⅘ = 3\ 13/2$ ly. Before this, already 5 ⅔ ly had passed. The total time passed since emitting the signal is thus 3 13/2 ly + 5 ⅔ ly = 9 5/24 ly.

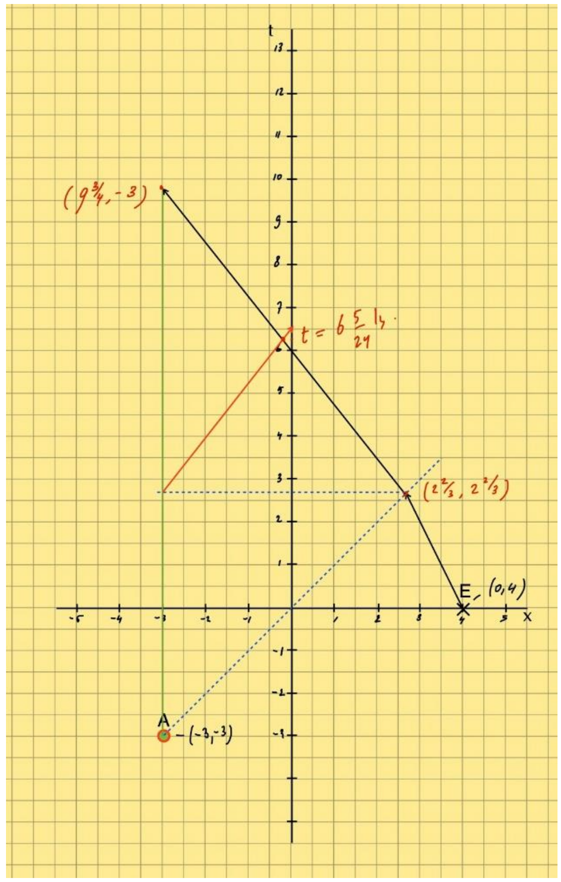


Figure 13 - Image for Question 17

Introduction for Question 18 - 20: Setanta (a mythical figure from Irish folklore) is sprinting towards a Hound at 3 m/s. At time t=0, the Hound is 30 m away from Setanta and is bounding towards him at 15 m/s (both speeds are relative to the ground frame). Consider the direction in which Setanta is travelling to be the +x direction in all frames. Setanta weighs 23 kg and the Hound weighs 70 kg.

18. What is the total momentum of the two in the ground frame?
   a. 981 kgm/s
   b. -981 kgm/s
   c. 1119 kgm/s
   d. -1119 kgm/s
   e. -1050 kgm/s

   Correct answer: B
   Explanation: Momentum is a vector, so we need to take the direction into account. Since Setanta is travelling in the +x direction and the Hound is travelling in the -x direction, that should be reflected in the sign of each of their velocities and thus their momenta.

   $\boldsymbol{\rho}_{total} = \boldsymbol{\rho}_{Setanta} + \boldsymbol{\rho}_{Hound}$
   $\boldsymbol{\rho}_{Setanta} = m_{Setanta} \times \mathbf{v}_{Setanta} = 23 \times 3 = 69$ kg m s^{-1}
   $\boldsymbol{\rho}_{Setanta} = m_{Setanta} \times \mathbf{v}_{Setanta} = 70 \times (-15) = -1050$ kg m s^{-1}
   $\boldsymbol{\rho}_{total} = 69 - 1050 = -981$ kg m s^{-1}

   The - sign is important! Because the Hound has a greater mass and a greater speed than Setanta, its contribution to the momentum is much greater and thus the total momentum is in the -x direction.

19. What is their total kinetic energy in the Hound's frame?
   a. 3726 J
   b. -3726 J
   c. 7978.5 J
   d. 7771.5 J
   e. -7771.5 J

   Correct answer: A
   Explanation: In the Hound's frame, the Hound itself is not moving, and Setanta is approaching the Hound with a velocity of 3 + 15 = 18 m s^{-1}. Since kinetic energy is ½ × mass × speed^2, in the Hound's frame the only kinetic energy is held by Setanta as he is the only one moving. In this frame:

   $E_{k,Setanta} = \frac{1}{2} \times m_{Setanta} \times (v_{Setanta})^2 = \frac{1}{2} \times 23 \times 18^2 = 3726$ J

   Because kinetic energy relies on the square of the velocity, its sign is always positive.

20. Setanta knows that the Hound will jump on him once he is 3 m away. How long will it take for this to happen if Setanta does nothing and keeps running?

~~a. 1.66 s~~
~~b. 1.68 s~~
~~c. 1.70 s~~
~~d. 1.72 s~~
~~e. 1.74 s~~

Correct answer: (Wasn't an option, will be accounted for)

Explanation: In Setanta's frame, he is not moving and the Hound is approaching him with a speed of 18 m s^{-1}. This makes Setanta's frame the easiest to work in for both this question and the next question. Now we only have one speed to work with, we can just divide the distance by the Hound's speed to get the time it takes for the Hound to jump on Setanta.

At t = 0 the Hound is 30 m away from Setanta. When the Hound is 3 m away he will jump on Setanta. This means the distance the Hound has to cover before he jumps on Setanta is 27 m. The time it takes for the Hound to cover this distance is 27 ÷ 18 = 1.5 s.

21. At t=0, Setanta hurls a ball at 18 m/s (in Setanta's frame) towards the Hound's head. How far away will the Hound be from Setanta when the ball hits him?

a. 12.5 m
b.13.6 m
c. 13.8 m
d. 15 m
e. 16.4 m

Correct answer: D

Explanation: We are once again going to work in Setanta's frame. In Setanta's frame, he is at rest, the Hound is approaching him with a velocity of -18 m s^{-1} and his ball is travelling towards the Hound with a velocity of 18 m s^{-1}. The ball will hit the Hound at x m away from Setanta. That meand that when the ball hits the Hound, the Hound will have travelled (30 - x) m.

$x \div 18 = (30 - x) \div 18$

In this case the two 18s cancel out.

$x = 30 - x$

$2x = 30$

$x = 15$ m

22. SN1987a is the remnant of a star that went supernova a long time ago. When a heavy star reaches the end of its lifetime, it collapses, building up the pressure in its core. This causes a big explosion - what we see as a very bright star in the sky. The light of this particular star reached us in 1987 (hence the name). The star is a distance of 1.586*10^18 km (51.4 kiloparsecs) away from us. When did the supernova occur?

a. 165 772 B.C.
b. 1819
c. 167 648 B.C.
d. 33
e. 1987

Correct answer: A.
Explanation: You can compute this through

$$t_{explosion} = 1987\ years - \frac{(1.586*10^{18} km)}{c} = -\ 165\ 772 = 165\ 772\ B.C.$$

23. Earth's humans and aliens from a spaceship trade materials near the earth. After a conflict, where the aliens stole some of our finest goods, they try to escape. Their ship leaves at a velocity of ½ in the +x direction. (Assume instant acceleration.) Earth launches a rocket at a velocity of ⅘ in the +x direction to take down the alien ship. When does light of the rocket taking down the ship arrive back at earth?

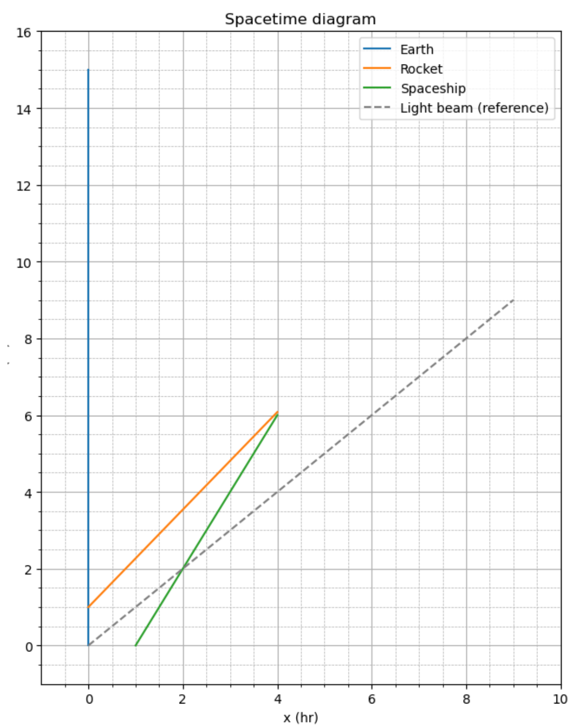


Figure 14 - Image for Question 23

a. t = 6 hr
b. t = 8 hr
c. t= 10 hr
d. t = 12 hr
e. t = 14 hr

Correct answer: C

Explanation: The light follows a line with a slope of -1 and traces to the left. For a line of slope with |-1|, it travels as much in the x direction as the t direction. It is 4 light hours away from the earth at the point of explosion, so it takes 4 hours for the light to travel back to earth. Add the 6 hours the rocket took to get to the spaceship, and we find t=10 hours, so answer C.

24. Earth's humans and aliens from a spaceship trade materials near the earth. After a conflict, where the aliens stole some of our finest goods, they try to escape. Their ship leaves at a velocity of ½ in the +x direction. (Assume instant acceleration.) Earth launches a rocket to take down the alien ship. What is the velocity of this rocket?

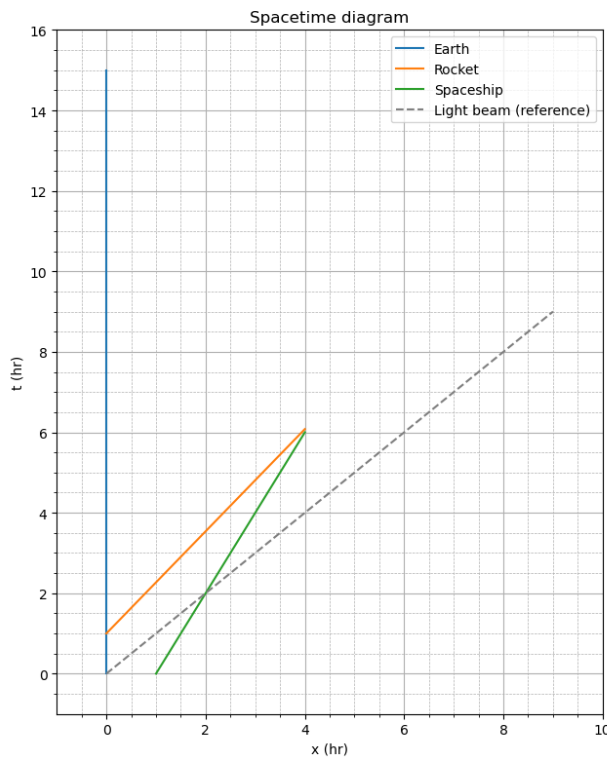


Figure 15 - Image for Question 24

a. 5/4
b. 1
c. 1/2
d. 4/5
e. 2

Correct answer: D
Explanation: The velocity of the rocket is proportional to the inverse of the slope of the rocket; we see that the rocket travels for 5 hours along the t-axis and 4 hours along the x-axis. The velocity of the rocket is hence v=4/5, answer D.

25. Using the radar method, we determine the distance from the earth to the moon to be 384 400 km. We shoot a laser at the moon at t=0. At what time do we observe its reflection?
a. 5.93 s
b. 2.564 s
c. 1.282 s
d. 0s
e. 3s

Correct answer: B
Explanation: The radar method lets a laser beam bounce of the moon such that we observe it's reflection later. A light beam takes 384 400km/c = 1.282 seconds to get to the moon. However, we also need to travel back from the moon, such that the total time travelled is 2.564 second. Answer B is hence correct.

26. SN1604 is the remnant of a star that went supernova a long time ago. When a heavy star reaches the end of its lifetime, it collapses, building up the pressure in its core. This causes a big explosion - what we see as a very bright star in the sky. The light of this particular star reached us in 1604 (hence the name). The star is a distance of 1.88*10^17 km (6.1 kiloparsecs) away from us. When did the supernova occur?

a. 18 300 B.C.
b. 1604
c. 17 900 B.C.
d. 59
e. 1584

Correct answer: A
Explanation: The time required for the light to travel from the supernova to earth is 1.88*10^17km/c = 6.28*10^11 seconds = 19 900 years. We subtract this from the year of observation and end up around 18 300 B.C.

27. Earth's humans and aliens from a spaceship trade materials near the earth. After a conflict, where the aliens stole some of our finest goods, they try to escape. Their ship leaves at a velocity of ½ in the +x direction. (Assume instant acceleration.) Earth launches a rocket at a velocity of ⅘ in the +x direction to take down the alien ship. What is the coordinate time difference between the rocket exploding and earth receiving the light of the explosion?

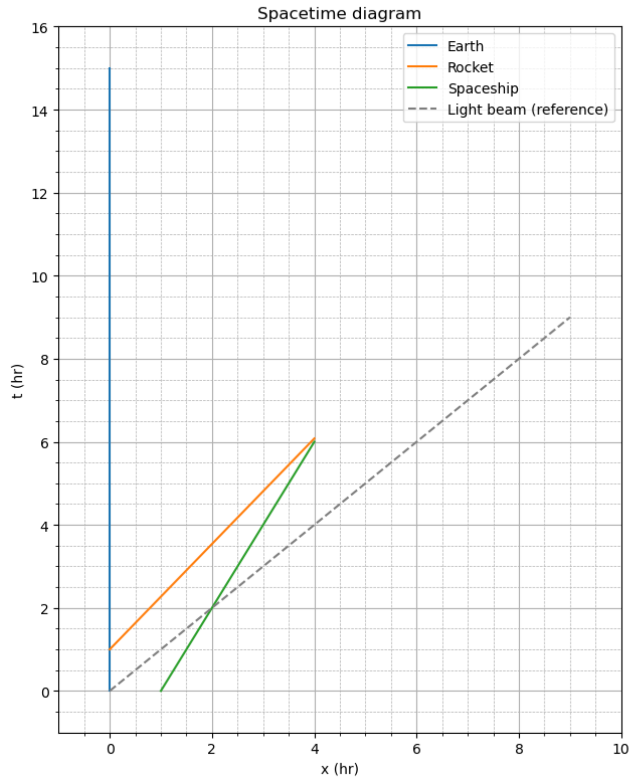


Figure 16 - Image for Question 27

a. t = 0 hr
b. t = 2 hr
c. t= 4 hr
d. t = 6 hr
e. t = 8 hr

Correct answer: C
Explanation: The coordinate time difference is the distance on the vertical axis, which in this case is 4 hours. Answer C is correct.

28. Earth's humans and aliens from a spaceship trade materials near the earth. After a conflict, where the aliens stole some of our finest goods, they try to escape. Their ship leaves in the +x direction. (Assume instant acceleration.) Earth launches a rocket to take down the alien ship. What is the velocity difference between the rocket and the spaceship?

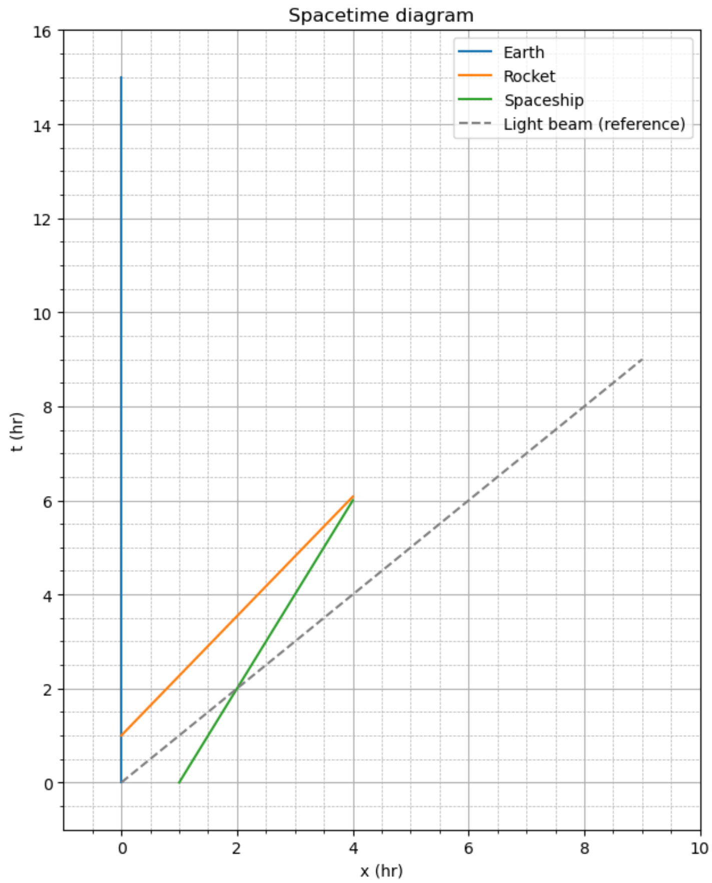


Figure 17 - Image for Question 28

a. 1/3
b. 4/5
c. 3/5
d. 3/10
e. 4/9

Correct answer: D
Explanation: The velocity of the objects can be determined by reading off the slopes of the objects and then inverting is. The slope of the rocket is 5/4, and the slope of the spaceship is 6/3 = 2. The inverse of the slopes (the velocities) hence are 4/5 for the rocket and 3/6 for the spaceship. Subtracting the fractions $\frac{4}{5} - \frac{3}{6} = \frac{24}{30} - \frac{15}{30} = \frac{9}{30} = \frac{3}{10}$ and hence the answer is D.

28. Using the radar method, we determine the distance from the earth to a comet to be 492 600 km. We shoot a laser at the comet at t=0. At what time do we observe its reflection?
a. 2.287 ms
b. 3.286 s
c. 1.643 s
d. 0s

e. 3s

Correct answer: B
Explanation: The time for light to travel a distance of 492 600 km can be computed by dividing it by the speed of light and gives t=1.643 seconds. We need the light to come back to us as well, such that the time between a round trip of the light is twice that. The total time is hence 3.286 seconds, answer B.

29. Suppose the Galilean transformation equations are true. The Muad'Dib is riding a Shai-hulud (a giant worm), gliding through the desert of Arrakis. As observed by some Fremen rebels standing still on the desert surface, the Shai-hulud is charging at a spice harvester at a velocity of 9*10^-8 in the -y direction. The spice harvester is racing away to safety at a velocity of 5*10^-8 with respect to the Fremen in the -y direction. What is the y-component of the Shai-hulud's velocity as measured by the spice harvester?
a. 1.4*10^-7
b. -1.4*10^-7
c. 4*10^-8
d. -4*10^-8
e. -9*10^-8

Correct answer: D
Explanation: The correct answer is D. Use

$$v'_{Shai-hulud} = v_{Shai-hulud} - \beta = -9 * 10^{-8} \text{ --- } 5 * 10^{-8} = -4 * 10^{-8}$$

where β is the velocity of the harvester's frame with respect to the ground frame.

30. The Muad'Dib has foreseen that a group of Sardaukar have set a trap for him. The Sardaukar, located a distance of 2 microseconds in the +x direction with respect to the Muad'Dib, fire a cannonball at the Shai-hulud in the -x direction at a velocity of 1.2*10^-7. What is the magnitude of the velocity of the cannonball as measured by the Muad'Dib?
a. 1.5*10^-7
b. 1.3*10^-7
c. 1.2*10^-7
d. 2.1*10^-7
e. 3*10^-8

Correct answer: A
Explanation: We have to take the magnitude of the cannonball's velocity, which is:

$$v'_{cannonball} = \sqrt{v'^2{}_x + v'^2{}_y} = \sqrt{(v^2{}_x + (v_y - \beta)^2} = \sqrt{(-1.2 * 10^{-7})^2 + (0 \text{ --- } 9 * 10^{-8})^2} = 1.5 * 10^{-7}$$

where β is the velocity of the Shai-hulud's frame with respect to the ground frame.

31. An intelligent microbe on the cannonball is observing the fate of the spice harvester. As measured by the microbe, what is the magnitude of the velocity of the spice harvester?
a. 1.2*10^-7

b. 5*10^-8
c. 7*10^-8
d. 1.7*10^-8
e. 1.3*10^-7

Correct answer: E
Explanation: The magnitude is:
$v'_{harvester} = \sqrt{v'^2_x + v'^2_y} = \sqrt{v^2_x + (v_y - \beta)^2} = \sqrt{(1.2 * 10^{-7})^2 + (0 -- 5 * 10^{-8})^2} = 1.3 * 10^{-7}$, where β is the velocity of the spice harvester's frame with respect to the ground frame.

32. The Muad'Dib has managed to evade the cannonfire, and is getting closer to the spice harvester. The spice harvester tries to fire a bullet at the Muad'Dib. The bullet leaves the spice harvester's gun at a velocity of 3*10^-7. What is the y-component of the velocity of the bullet as measured by the Muad'Dib?
a. 2.1*10^-7
b. -3.9*10^-7
c. 3.9*10^-7
d. 3.4*10^-7
e. -3.4*10^-7

Correct answer: D
Explanation: The velocity of the bullet as measured in the ground frame is given as vbullet=(v'bullet+β) = 3*10-7+-5*10-8=2.5*10-7
The velocity of the bullet as measured in the Muad'Dib's frame is:
v''bullet=(vbullet-β') = 2.5*10-7--9*10-8=3.4*10-7, where β is the velocity of the spice harvester's frame with respect to the ground frame, and β'is the velocity of the Muad'Dib's frame with respect to the ground frame.

33. A car is moving in the +x direction with a constant velocity of 30 m/s. A cyclist is moving nearby in the +y direction with a constant velocity of 8 m/s. According to the driver, what is the velocity of the cyclist in the x direction?
a. -31 m/s
b. -30 m/s
c. 0 m/s
d. 8 m/s
e. 30 m/s

Correct answer: B
Explanation: Both observers are moving perpendicular to each other, they both have no velocity in the direction of the other observer. The driver is moving away from the cyclist with a velocity of 30 m/s in the x direction. The cyclist has no x velocity component, so the driver sees the cyclist moving with a velocity of -30 m/s, since it looks like the cyclist is moving backwards from the point of view of the driver.

34. A cyclist traveling with a velocity of 10 m/s in the +x direction passes a hiker walking with a velocity of 1.5 m/s in the same direction. According to the hiker, what is the velocity of the cyclist in the x direction?
a. - 10 m/s
b. -8.5 m/s
c. 1.5 m/s
d. 8.5 m/s
e. 10 m/s

Correct answer: D
Explanation: The cyclist goes faster than the hiker, so the hiker observes that the cyclist has a positive velocity in the x direction. Since the hiker has a velocity of 1.5 m/s in the same direction as the cyclist, the hiker will observe that the cyclist moves a bit slower than an observer that does not move would see. The velocity of the cyclist that the hiker will measure is 10 - 1.5 = 8.5 m/s.

35. The distance between Earth and the asteroid belt is 35 minutes (in SR units). An astronaut stands on an asteroid in the belt with a clock that is synchronized with the clock of an observer on Earth. When the clock of the astronaut shows 3 minutes he waves to the observer on Earth. What time on the clock does the observer on Earth measure when he sees the astronaut waving? Hint: draw a spacetime diagram.
a. 3 minutes
b. 35 minutes
c. 38 minutes
d. 70 minutes
e. 73 minutes

Correct answer: C
Explanation: Since it takes light 35 minutes to travel a distance of 35 minutes, the observers on Earth will see anything that happens in the asteroid belt with a delay of 35 minutes. So if something happens at 3 minutes according to the synchronized clock, the observers on Earth will **see** it at 38 minutes. Of course they can deduce (observe) that it happened at 3 minutes, but the question asks when they would see it.

36. A spaceship moves with a constant velocity away from Earth. When the spaceship is 10 s (in SR units) away from Earth, it shoots a light flash to a mirror on Earth. The light flash returns to the spaceship 24 seconds later. What is the difference between the time at which the light flash hits the mirror in both inertial frames? Hint: draw a spacetime diagram.
a. 0 s
b. 2 s
c. 7 s
d. 14 s

e. 24 s

Correct answer: B

Explanation: This question can be answered using the radar method. The radar method allows us to calculate the time at which the light flash hits the mirror in the frame of the spaceship. Since the difference between emission and reception is 24 seconds, the time at which the light flash hit the mirror is 12 seconds. In Earth's frame, the light flash would arrive at 10 s, because the spaceship is 10 s away from Earth when it emits the light flash. The difference between the reflection time in both frames is therefore 2 seconds.